Switched Beam Antenna Design Principles for Angle of Arrival Estimation

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***Abstract*— Switched Beam Antennas support radio positioning via Angle Of Arrival (AOA) information collected from nearby devices. Using an analytical approach, first we present the Cramér–Rao Bound (CRB) for AOA estimates using identically and equally spaced antenna elements. Then we analyze the results to devise design guidelines for improved AOA estimation. The design parameters considered are: 1) the number of antenna elements, 2) their directivity, and 3) the type of polarization. The effect of each parameter is discussed in detail; additionally, experimental results at 2.45 GHz are reported to evaluate the effect of different antenna polarization on the CRB.**

***Index Terms*— Wireless Networks, Switched Beam Antenna, Design, Radio-Positioning, Angle of Arrival Estimation, CRB.**

I. Introduction

Radio-positioning systems support *location-awareness* in applications where the use of GPS is not cost effective or technically feasible (e.g. indoor) [1], [2]. Among the many solutions proposed, systems based on *Angle Of Arrival* (AOA) estimation have several qualities that make them attractive to a wide range of applications. AOA localization implements *fine-grained* positioning without requiring knowledge of the propagation model for the RF signal. Additionally, angle-based positioning only requires measurements from two anchor nodes (see Fig. 1), and in a recent work we have demonstrated an AOA system capable of single-anchor localization [3].

AOA estimation requires augmenting one of more Base Stations (BS) with smart antennas capable of generating multiple beams. A common solution consists in combining patch antennas pointing in different directions. But while this approach is easy to implement, to the best of our knowledge, design principles that help in implementing effective AOA positioning system have not been discussed in the literature.

The goal of this work is to evaluate how different antenna parameters will affect the capacity of the system to accurately estimate AOA information. Section II introduces the basis of the estimation process using directional antennas. After having defined the measurement model for the AOA estimates, Section III presents an analytical expression for the theoretical limit that bounds the estimation accuracy.

In Section IV, analysis of the Fisher information and the CRB is used to evaluate the effect of some important design parameters when using a system with identical and equally spaced antenna elements. The parameters considered are: 1) the number of antenna elements, 2) the directivity of each element, and 3) the antenna polarization. Results of our analysis shows that while increasing the number of antennas

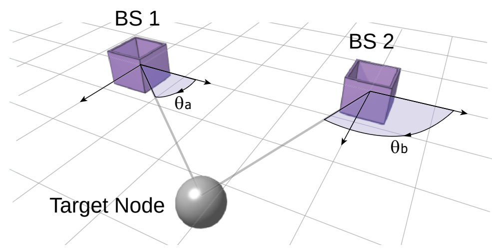


Fig. 1: 2D target positioning using AOA measurements from two Base Stations (BS) with four patch antenna elements.

always results in improved estimation accuracy, increasing the directivity of each element could lead to counterproductive results. Additionally, experimental results show the effect of different antenna polarization on the estimation error. Section V concludes the paper using the CRB analysis to evaluate the AOA estimation error for an antenna prototype with radiation patterns measured in the anechoic chamber.

II. Angle Of Arrival Estimation

A. Measurement Model

Consider a BS equipped with N_r selectable antenna beams. The system is designed to estimate the AOA of the messages transmitted by a target node located at an arbitrary position. To this purpose, the BS measures the *Received Signal Strength* (RSS) on each face i.

Assuming values measured in dB or dBm, the RSS on each face i can be written as:

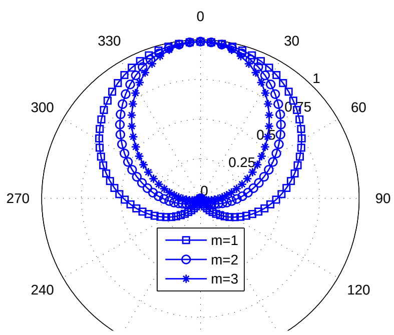
$$S_i = G_i(\theta) + P_\mathrm{rx} + v_i, \tag{1}$$

where $G_i(\theta)$ is the gain (in dB) of each antenna in the direction of the target, P_rx is the RF power arriving at the BS, and v_i is a random component that models the noise. Note that each measurement S_i is a random variable that depends on the unknown parameters θ and P_rx.

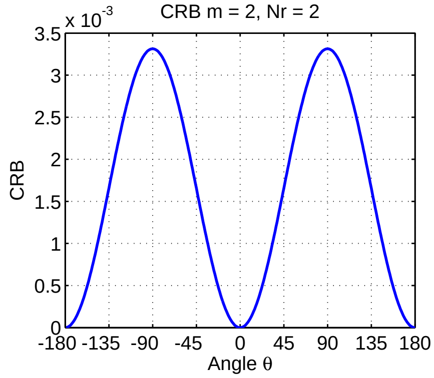
The P_rx value varies with the transmission power used by the target and with the random attenuation introduced by the radio channel. Since P_rx does not carry information on θ, its contribution can be removed by considering the RSS differences between two faces i and j:

$$S_{i,j} = S_i - S_j = G_i(\theta) - G_j(\theta) + n_{ij}, \tag{2}$$

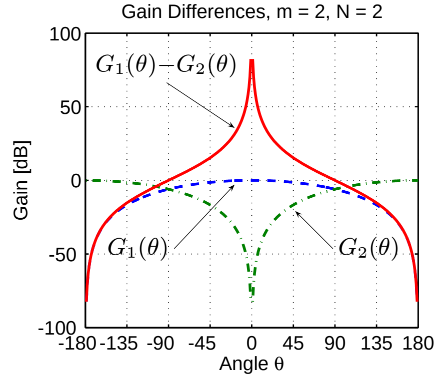
where $n_{ij} = v_i - v_j$. In addition, if the system is built with N_r identical and equally spaced antenna elements, and if the target's distance is much larger than distance between the



(a)



(b)



(c)

Fig. 2: **a)** Cardioid patterns for various m; **b)** CRB for a system with two antennas; **c)** Gain differences between two antennas.

faces, equations (2) can be simplified by expressing the gains G_i's as a function of a common radiation pattern G:

$$G_i(\theta) = G\left(\theta - i\frac{360^\circ}{N_\text{r}}\right). \tag{3}$$

B. Fisher Information Theory

While several algorithms for AOA estimation are available using the measurements S_i or S_{ij} (e.g. MUSIC [4]), our goal is to obtain general design guidelines for the antennas used at the BS. Analysis in this section adopts the Fisher information (F) [5], [6] to evaluate the accuracy of a generic AOA estimation algorithm in relation to some important design parameters.

The value F measures the amount of information that a random variable carries about an unknown parameter. In the case discussed, the random variables are the S_{ij} values, while the parameter to estimate is θ. The Fisher information is:

$$F_{ij}(\theta) = E\left\{\left[\frac{\partial}{\partial\theta}\log f_{ij}(s_{ij};\theta)\right]^2\right\}, \tag{4}$$

where $f_{ij}(s_{ij};\theta)$ is the *probability density function* (pdf) that describes the measurements S_{ij}. Assuming *independent and identically distributed* (i.i.d) noise components n_{ij} with Gaussian distribution $\mathcal{N}(0,\sigma^2_\text{rss})$, it is possible to demonstrate that the total amount of Fisher information available from a system with N_r antennas is:

$$\begin{aligned} F(\theta) &= \frac{1}{\sigma^2_\text{rss}}\sum_{\{i,j\}\in\mathcal{C}}\left[\frac{\partial\left(G_i - G_j\right)}{\partial\theta}\right]^2 = \\ &= \frac{1}{\sigma^2_\text{rss}}\left[N_r\sum_{i=0}^{N_\text{r}-1}\left(\frac{\partial G_i}{\partial\theta}\right)^2 - \left(\sum_{i=0}^{N_\text{r}-1}\frac{\partial G_i}{\partial\theta}\right)^2\right], \end{aligned} \tag{5}$$

where $\mathcal{C}$ denote the set of the $N_\text{r}(N_\text{r}-1)/2$ distinct antenna pairs $\{i,j\}$.

III. AOA Estimation Bound

The Fisher information $F(\theta)$ expresses the theoretical limit achievable when estimating θ. If T is an unbiased estimator for θ, the inverse of $F(\theta)$ bounds the minimum variance of the estimation error:

$$\text{Var}\left[T(X)\right] \geq F^{-1}(\theta). \tag{6}$$

The inequality (8), known as the *Cramér-Rao Bound* (CRB) [5], is a limit that applies to *any* estimator that uses measurements S_{ij} to compute θ.

The Fisher information and the CRB depends on the gains G_i of the antennas used in the system. To obtain general design guidelines, we model the gain of a generic patch antenna using a *cardioid* shaped function with exponent $m \geq 1$, and maximum gain G_max:

$$G(\theta) = G_\text{max}\left(\frac{1+\cos(\theta)}{2}\right)^m = G_\text{max}\left(\cos(\theta/2)\right)^{2m}. \tag{7}$$

As shown in Fig. 2a, larger values of the exponent m correspond to more directive antennas. Substituting (7) into (5), we obtain an analytical expression for the CRB:

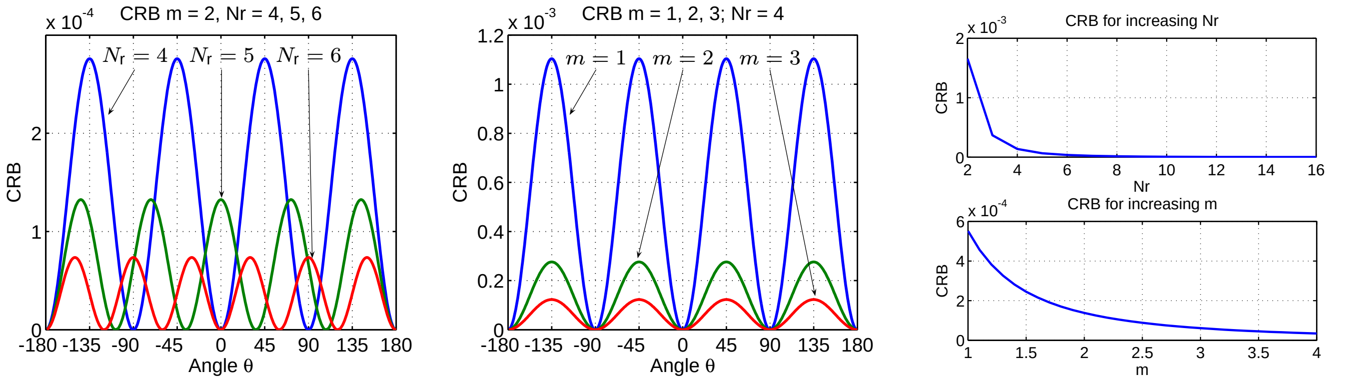
$$\text{CRB}(\theta) = \frac{m^{-2}\sigma^2_\text{rss}}{N_r\sum_{i=0}^{N_r-1}\tan\left(\frac{\theta-i\Delta}{2}\right)^2 - \left(\sum_{i=0}^{N_r-1}\tan\left(\frac{\theta-i\Delta}{2}\right)\right)^2}, \tag{8}$$

where $\text{CRB}(\theta) = F^{-1}(\theta)$, and $\Delta = 360^\circ/N_\text{r}$.

Figure 2b shows the CRB for a system with $N_\text{r} = 2$ faces and $m = 2$. Note the different values of the CRB for different θ values. The minimum error is achieved for angles $\theta = 0^\circ$ and 180°, while larger estimation error are to be expected for angles close to $\theta = \pm 90^\circ$. The error can be understood by observing the function that describes the gain difference between two equispaced faces (see Fig. 2c). The function $G_1(\theta) - G_2(\theta)$ changes abruptly for angles close to 0° and 180°: small variations of the angle determine large variation in the gains' difference. Since the Fisher information depends on the derivative of $G_1(\theta) - G_2(\theta)$, this condition corresponds to a lower estimation error for θ. On the other hand, the estimation error increases for angles close $\pm 90^\circ$, where $G_1(\theta) - G_2(\theta)$ has a less steep slope.

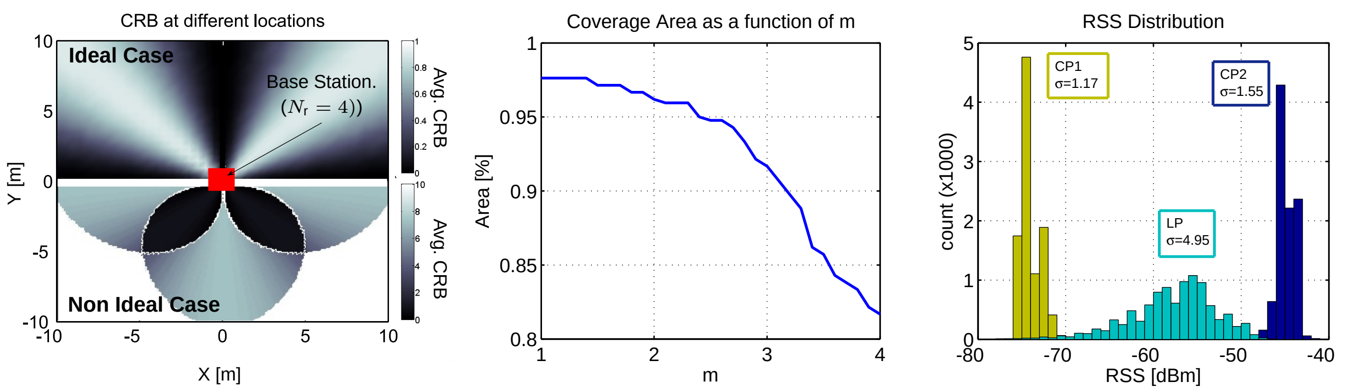
IV. Antenna Design Principles

Equation (8) shows that the CRB depends on two fundamental antenna design parameters: 1) the number of antenna faces N_r, and 2) the exponent m that controls the directivity. Another parameter of fundamental importance is the RSS measurement variance σ^2_rss. Although σ^2_rss largely depends on the radio channel, a proper choice of the antenna polarization can mitigate the effect of multipath propagation and improve AOA estimation.



(a) (b) (c)

Fig. 3: **a)** CRB as function of θ for increasing number of antennas N_r ($\sigma^2_{\mathrm{rss}} = 1\,\mathrm{dB}$); **b)** CRB as a function of θ for increasing exponent values m ($\sigma^2_{\mathrm{rss}} = 1\,\mathrm{dB}$). **c)** Average CRB as a function of N_r (top), and m (bottom).



(a) (b) (c)

Fig. 4: **a)** Comparison between the CRB for ideal transceivers (top) and devices with finite sensitivity (bottom). The CRBs are computed for a system with $N_\mathrm{r} = 4, m\ = 1$ operating in an $10 \times 10\,\mathrm{m}$ square area; the RSS at various distances is simulated as $P_r(d) = -45 - 10n_p \log_{10}(d)$, with $n_p = 2.5$; **b)** Coverage area for finite sensitivity receiver ($P_s = -95\,\mathrm{dBm}$) with $N_\mathrm{r} = 4$ and increasing m; **c)** Experimental RSS variance measured in a indoor experiment with different antennas: (LP = linear polarization., CP = circular polarization).

The following sections explore the effects of these parameters and provide guidelines to reduce the estimation error.

A. Number of Antenna Elements N_r

The parameter N_r directly affects the CRB. A larger number of antenna elements correspond to a larger number of independent measurements S_{ij}; therefore, increasing N_r always has a positive impact on the performance of a system for AOA estimation. Figure 3a shows the CRB as a function of θ for $N_\mathrm{r} = \{4, 5, 6\}$. Increasing N_r reduces the absolute value of the CRB maxima and increases the number of points where the error has its minimum.

Figure 3c(top) shows that increasing N_r steadily decreases the average value of the CRB. Thus, from an antenna design point of view, the maximum number of antennas should be limited only by cost constraints and the complexity of the feed and control circuitry necessary to support multiple patches.

B. Directivity Coefficient m

In principle, increasing the directivity of each antenna also increases the estimation accuracy of the system. Figures 3b and 3c(bottom) show the CRB for different values of the exponent m used to model the antenna gains in (7). In practice, however, the choice of the parameter m should be evaluated by taking into account the sensitivity of the transceiver(s) connected to the antenna elements. When using COTS radio packet transceivers (e.g. IEEE 802.x.y), the RSS measurements are only available if the messages are received with a power that exceeds the radio's sensitivity P_s. If this condition is not met, the probability of error rapidly increases, and the messages will likely be lost without producing valid RSS measurements. Therefore using antennas with high directivity can potentially reduce the coverage area of the system and result in a loss of performance due missing measurements on some of the antenna faces. If the number of faces capable of receiving a message drops below two, no AOA estimation is possible.

Figure 4a illustrates this concept by showing the difference between use of idealized radio communication and use of a transceiver with finite sensitivity. Consider a BS with $N_\mathrm{r} = 4$ and $m = 1$ deployed at the center of a square area measuring $20\,\mathrm{m} \times 20\,\mathrm{m}$. The top half of the plot shows the CRB for an ideal radio system. Since the transceiver sensitivity is assumed to be infinite, the CRB at different locations only depends on the AOA of the target. The bottom half of Fig. 4a shows the effect of using a transceiver with limited sensitivity. As the target moves away from the BS, the CRB increases due to messages lost on some of the faces.

Figure 4b summarizes the results relative to the example in Fig. 4a. Note that increasing m excessively reduces the coverage area of the system, i.e. the locations where AOA estimation is possible.

C. Antenna Polarization

In equation (8), the RSS variance σ^2_{rss} multiplies the CRB; therefore reducing σ^2_{rss} will directly reduce the uncertainty in the AOA estimates.

While the radio channel is the primary source of measurement dispersion due to time-varying multipath propagation and channel noise, the term σ^2_{rss} can be reduced by a proper choice of antenna polarization. Previous work has demonstrated that antennas operating in *circular polarization* (CP) are effective in reducing the measurements variance σ^2_{rss}. Given their capability to mitigate multi-path propagation, CP antennas have been exploited in wireless system operating indoors [7], and in radio-positioning applications [8].

To quantify the effect of different antenna choices, we report the results of RSS measurements in an industrial indoor environment using $2.45\,\text{GHz}$ communication. The measurements were generated using a target node equipped with a monopole antenna. At the other end of the radio link, the RSS values were measured by multiple BS implementing different types of antenna. Figure 4c shows the RSS distribution for a printed patch antenna working in linear polarization (LP), and two antennas working in circular polarization: an antenna with directivity equal to $8.5\,\text{dBm}$ (CP1), and one with lower directivity, $6.4\,\text{dBm}$, but higher gain (CP2). Note the larger variability of the measurements using linear polarization: the standard deviation σ_{rss} is about four time larger than the value measured using circular polarization. In this application scenario, using CP antennas would reduce the CRB by a factor 16 with respect to antennas with linear polarization.

V. Calculation of the CRB from Experimental Switched Beam Antenna Pattern

Section IV has presented analytical results based on antenna elements with an idealized gain $G(\cdot)$. Analysis of antennas with an arbitrary radiation pattern is possible by numerical differentiation of G in (5).

Figure 5 shows the radiation patterns of a switched-beam antenna prototype with 4 patch antennas arranged to form a cube. Although the patches were manufactured using a common design, some differences in the radiation pattern are visible. The CRB computed as a function of the angle θ is reported on the bottom part of Figure 5. As a result of the irregularities in the radiation patterns, the CRB profile is quite different from the ideal case. Despite these differences, the proposed CRB approach could be used to alter the antenna design and remove the large error in correspondence of some angles θ. The general guidelines presented in Section IV remain valid to improve the AOA estimation accuracy, for example by increasing the number of antenna elements or by using circular polarization.

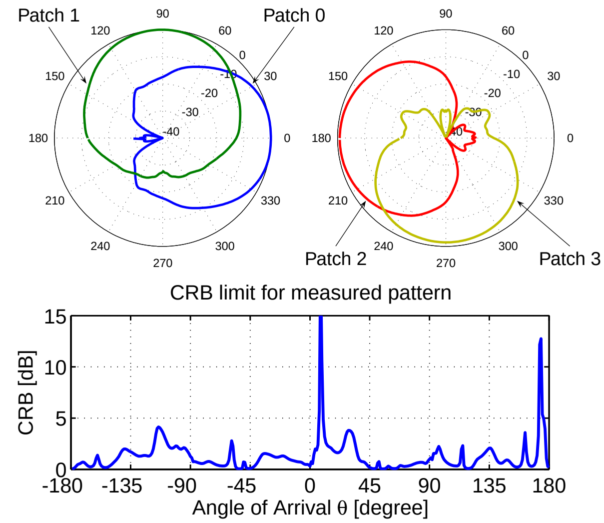


Fig. 5: (**top**) Normalized Patterns of the four antennas (the normalization is referred to the most directive). (**bottom**) Variance computed with the Cramér–Rao Bound based on the actual antenna gains.

VI. Conclusions

Using an information-theoretical approach similar to the one that we previously adopted to compare range-free and range-based localization [9], we have presented the Fisher information and the CRB for AOA estimation using a system with N_{r} identical and equally spaced patch antennas. Analysis of the results has highlighted the effect of some important design parameters on the system performance. In particular, the minimum uncertainty on the AOA estimates has been shown to be related to: the number of antenna elements, their directivity and the polarization used. The proposed analysis has also taken into account the effect of the limited receiver sensitivity on practical systems for AOA estimation. Finally, we have shown the CRB for an antenna with non ideal patterns.

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